

Guided Examples: The Remainder and Factor Theorems

Factors of Polynomials The Factor Theorem can help you find all the factors of a polynomial.

Factor Theorem	The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.
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Example: Show that $x + 5$ is a factor of $x^3 + 2x^2 - 13x + 10$. Then find the remaining factors of the polynomial.

By the Factor Theorem, the binomial $x + 5$ is a factor of the polynomial if -5 is a zero of the polynomial function.

To check this, use synthetic substitution.

$$\begin{array}{r|rrrr}
 -5 & 1 & 2 & -13 & 10 \\
 & & -5 & 15 & -10 \\
 \hline
 & 1 & -3 & 2 & 0
 \end{array}$$

Since the remainder is 0, $x + 5$ is a factor of the polynomial. The polynomial $x^3 + 2x^2 - 13x + 10$ can be factored as

$(x + 5)(x^2 - 3x + 2)$. The depressed polynomial $x^2 - 3x + 2$ can be factored as $(x - 2)(x - 1)$.

So $x^3 + 2x^2 - 13x + 10 = (x + 5)(x - 2)(x - 1)$.

WE DO: Exercises

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

1. $x^3 + x^2 - 10x + 8; x - 2$

2. $x^3 - 4x^2 - 11x + 30; x + 3$

3. $x^3 + 15x^2 + 71x + 105; x + 7$

4. $x^3 - 7x^2 - 26x + 72; x + 4$

You Do:

5. $2x^3 - x^2 - 7x + 6; x - 1$

6. $3x^3 - x^2 - 62x - 40; x + 4$

7. $12x^3 - 71x^2 + 57x - 10; x - 5$

8. $14x^3 + x^2 - 24x + 9; x - 1$

9. $x^3 + x + 10; x + 2$

10. $2x^3 - 11x^2 + 19x - 28; x - 4$

11. $3x^3 - 13x^2 - 34x + 24; x - 6$

12. $x^4 + x^3 - 11x^2 - 9x + 18; x - 1$